Industrial Automation and Robotics

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SOLUTION

Industrial automation and robotics
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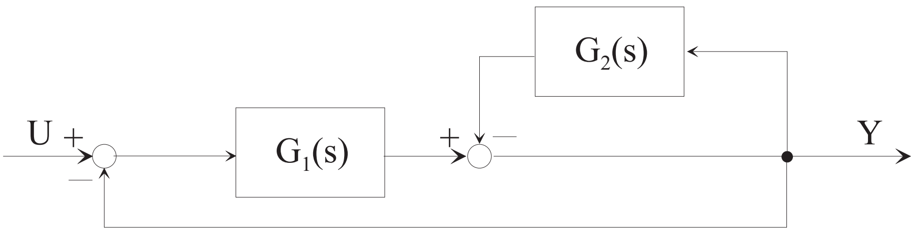
EXERCISE 1

1. Consider the dynamic system described by the following block diagram:



Solve the block diagram by determining the transfer function from u to y.

The block diagram includes an inner feedback loop with unitary forward transfer function and negative feedback with transfer function G_2, whose closed-loop transfer function is $\frac{1}{1+G_2}$. In turn, this system, in series with G_1, is closed in another unitary negative feedback. The resulting transfer function is:

$$\frac{Y}{U} = \frac{\frac{G_1}{1+G_2}}{1 + \frac{G_1}{1+G_2}} = \frac{G_1}{1 + G_1 + G_2}$$

2. Discuss whether it is necessary and/or sufficient that one or more of the transfer functions be asymptotically stable in order for the overall system to be asymptotically stable

Since both G_1 and G_2 are closed in feedback loops, it is neither necessary nor sufficient that any of them be asymptotically stable for the overall system to be asymptotically stable.

3. Setting $G_1(s) = \dfrac{1}{1+s}$, $G_2(s) = k$, assign the parameter k such that the dc gain of the overall transfer function is $\mu = 0.25$.

Substituting the expressions of G_1 and G_2 we obtain:

$$\frac{Y}{U} = G(s) = \frac{\frac{1}{1+s}}{1 + \frac{1}{1+s} + k} = \frac{1}{2 + k + (1+k)s}$$

The dc gain of this transfer function is the value that it takes for $s = 0$:

$$\mu = G(0) = \frac{1}{2+k}$$

which is equal to 0.25 when $k = 2$. In this case the transfer function is:

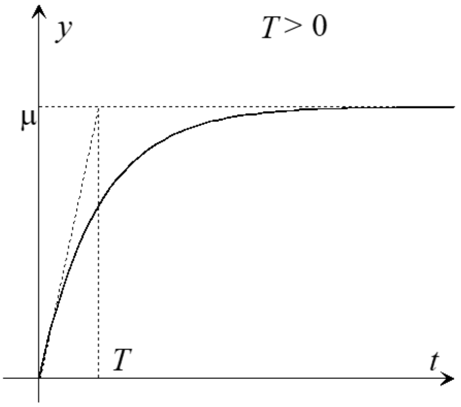
$$\frac{Y}{U} = G(s) = \frac{1}{3s + 4}$$

4. Using the value of k found at the previous step, sketch the step response of the transfer function from U to Y. What is the approximate time for the output to practically reach the steady state value?

The transfer function can be written in the form:

$$\frac{Y}{U} = G(s) = \frac{\mu}{1 + Ts}$$

with $\mu = 0.25$ and $T = 0.75$. The step response can be easily sketched:



The steady state is reached after approximately 5 times the time constant T, i.e. after $3.75s$.

EXERCISE 2

1. Consider the Ladder Diagram programming language for PLCs. List the types of timers that can be included in a Ladder Diagram, briefly explaining how they work.

Two types of timers can be used:

- The ***normal*** timer: it counts the time as long as there is electrical continuity at its left. After the duration indicated in the symbol of the timer has elapsed, the associated variable is set to 1. This variable is reset to 0 whenever there is no electrical continuity at the left of the timer, in which case the count of time restarts from zero.
- The ***latch*** timer: it counts the time as long as there is electrical continuity at its left. After the duration indicated in the symbol of the timer has elapsed, the associated variable is set to 1. This variable is reset to 0 with an appropriate reset coil. If there is no electrical continuity at the left of the timer, the count of time is suspended.

2. Consider now the following process: pressing a button **START** a light turns on for one minute. After such time interval, the light turns off and for 10 minutes pressing the button **START** cannot turn on the light, unless a second button **RESTART** is first pressed (pressing the button when the light is on has no effect). Program the system with a Ladder Diagram code.

   Apart from the inputs **START** and **RESTART** and an output **LIGHT**, we can use two timers with associated variables **T1** and **T2** and an internal variable **CONS**. A possible Ladder diagram is reported hereafter:

```
    START     T1      T2    CONS
|----| |-----|/|-----|/|-----(L)-----|
|                                    |
|                                    |
|   CONS         T1        LIGHT     |
|----| |-----+--|/|--------( )-------|
|            |                       |
|            |                       |
|            |   +-----------+       |
|            +---| T1        |-------|
|                | 1m        |       |
|                +-----------+       |
|                                    |
|     T1         +-----------+       |
|----| |---------| T2        |-------|
|                | 10m       |       |
|                +-----------+       |
|                                    |
|     T2              CONS           |
|----| |----------+---(U)------------|
|                 |                  |
|                 |                  |
|  RESTART  T1    |                  |
|----| |----| |---+                  |
|                                    |
```

3. Explain what is the ISO-OSI communication protocol and mention the names of the levels 1 and 2.

   The ISO-OSI is a conceptual model that standardizes the functions of a communication network, without regard to their underlying internal structure and technology. The model organizes a communication system into seven abstraction layers. A layer serves the layer above it and is served by the layer below it. The first two layers are the physical layer and the data link layer.

4. Consider now the Ethernet protocol for the digital communication on a bus: briefly describe how the access to the bus among the various agents is handled.

Ethernet adopts Carrier Sense Multiple Access with Collision Detection (CSMA/CD): if the medium is idle any node can start a transmission at any time. The transmitter monitors for collisions during transmission and, if a collision is detected, the frame is transmitted again. This protocol is not suitable for real-time applications as no bound on the transmission delay exists.

EXERCISE 3

1. In the framework of the motion planning for a robot, explain what is the meaning of an instruction in the PDL2 programming language `MOVE LINEAR TO POS WITH $LIN_SPD=0.6`

   The instruction refers to a motion command in the operational space, where the end effector moves along a linear path (a segment) towards the final destination POS, with linear velocity equal to $0.6m/s$.

2. Consider now the design of a trajectory for a variable q with a cubic time law. The total displacement is $h = 2$, the total positioning time is $T = 2s$ and the velocity at the beginning and at the and are zero. Compute the expression of variable q and of its first time derivative $\dot{q}$.

   The cubic polynomial for the variable q can be written as:

$$q(t) = a_0 + a_1 t + a_2 t^2 + a_3 t^3$$

   with boundary conditions $q(0) = 0$, $q(2) = 2$.
   The second order polynomial for the speed can be written as:

$$\dot{q}(t) = a_1 + 2a_2 t + 3a_3 t^2$$

   with boundary conditions $\dot{q}(0) = 0$, $\dot{q}(2) = 0$.
   The boundary conditions at time $t = 0$ yield $a_0 = a_1 = 0$, the boundary conditions at time $t = 2$ yield the system:

$$\begin{aligned} 4a_2 + 8a_3 &= 2 \\ 4a_2 + 12a_3 &= 0 \end{aligned}$$

   whose solution is $a_2 = 1.5$ and $a_3 = -0.5$. Polynomials for position and speed can then be written as:

$$\begin{aligned} q(t) &= 1.5t^2 - 0.5t^3 \\ \dot{q}(t) &= 3t - 1.5t^2 \end{aligned}$$

3. Consider the generation of the trajectory in the operational space. Write the expression of a segment (linear Cartesian path) for the position of the end effector.

   The expression of the segment is as follows:

$$\mathbf{p}(s) = \mathbf{p}_1 + \frac{s}{\|\mathbf{p}_2 - \mathbf{p}_1\|}(\mathbf{p}_2 - \mathbf{p}_1)$$

4. Assume now that the initial point for the end effector is $\mathbf{p}_1 = \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix}$ and the final point is $\mathbf{p}_2 = \begin{bmatrix} \sqrt{2} \\ 0 \\ 3 \end{bmatrix}$. Suppose that you want to use the cubic profile previously computed for the generation of the operational space trajectory (the segment as previously discussed). Explain if and how this can be done. What would be the maximum linear velocity of the end-effector in this case?

The time law is assigned to the natural coordinate s:

$$\mathbf{p}(s) = \mathbf{p}_1 + \frac{s(t)}{\|\mathbf{p}_2 - \mathbf{p}_1\|}(\mathbf{p}_2 - \mathbf{p}_1)$$

Since the displacement to cover is:

$$h = \|\mathbf{p}_2 - \mathbf{p}_1\| = \| \begin{bmatrix} \sqrt{2} \\ -1 \\ 1 \end{bmatrix} \| = \sqrt{2+1+1} = 2$$

we can use $s(t) = q(t)$. The maximum velocity of the end effector is the maximum speed achieved by the time law, in our case taken in the middle of the trajectory:

$$\dot{s}_{max} = \dot{s}(1) = 1.5$$