Industrial Automation and Robotics

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SOLUTION

INDUSTRIAL AUTOMATION AND ROBOTICS
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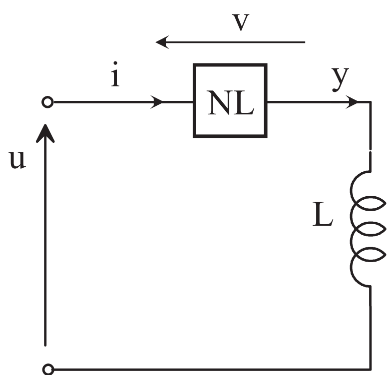
EXERCISE 1

Consider the electrical network sketched in the picture:



where the nonlinear element NL enforces the following relation between the current i passing through it and the corresponding voltage v across it:

$$v = i^3$$

1. Write the equations of the dynamic system that describes the electrical network. Then, setting $L = 1$, identify the equilibrium point corresponding to the constant input $u = \bar{u} = 8$

   If x is the current in the circuit, the voltage on the nonlinear element is x^3 while the voltage on the inductor is $L\dot{x}$. The balance of the voltage is therefore:

   $$u = x^3 + L\dot{x}$$

   The equations of the dynamic system are:

   $$\begin{aligned} \dot{x} &= -\frac{1}{L}x^3 + \frac{1}{L}u \\ y &= x \end{aligned}$$

   The equilibrium state is obtained setting the derivative to zero:

   $$-\frac{1}{L}\bar{x}^3 + \frac{1}{L}\bar{u} = 0$$

   The only possible solution is $\bar{x} = 2$.

2. Write the equations of the linearized system around the equilibrium state previously obtained and derive the expression of the corresponding transfer function.

The linearized system is:

$$
\begin{aligned}
\dot{\delta x} &= -3\frac{1}{L}\bar{x}^2\delta x + \frac{1}{L}\delta u \\
\delta y &= \delta x
\end{aligned}
$$

and then:

$$
\begin{aligned}
\dot{\delta x} &= -12\delta x + \delta u \\
\delta y &= \delta x
\end{aligned}
$$

Moving to the “s-domain”, we have:

$$
\begin{aligned}
s\delta X &= -12\delta X + \delta U \\
\delta Y &= \delta X
\end{aligned}
$$

The transfer function is then easily obtained as:

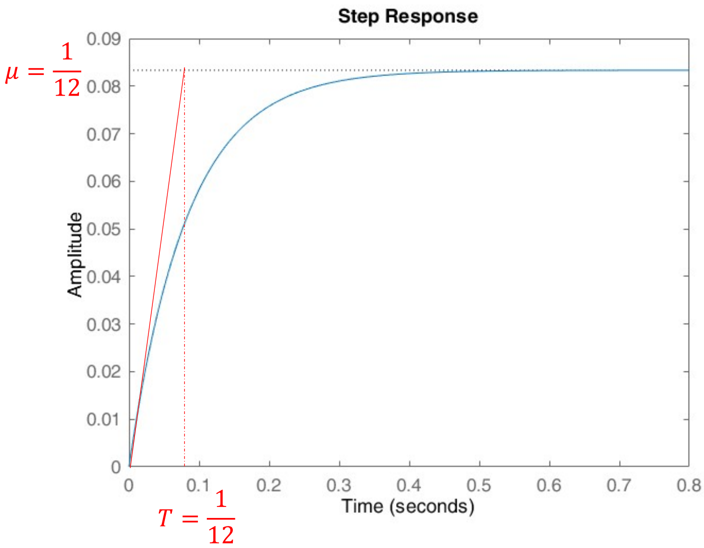
$$G(s) = \frac{Y}{U} = \frac{1}{s+12}$$

3. Sketch the step response of the linearized system previously obtained.

The transfer function can be expressed in the following form:

$$G(s) = \frac{\mu}{1+sT}$$

with $\mu = \frac{1}{12}$ and $T = \frac{1}{12}$.
The step response can be easily sketched, as in the following picture:



4. For the linearized system, suppose that an input $\delta u(t) = \sin(t)$ is assigned. Without going through the mathematical computations, explain how the expression of the output, after an initial transient, can be computed.

   Since the system is asymptotically stable, the following formula can be used:

$$\delta y(t) = \|G(j)\| \sin\left(t + \angle G(j)\right)$$

   where $G(j)$ is the frequency response at frequency $\omega = 1$ of the system.

EXERCISE 2

1. Consider the Ladder Diagram programming language for PLCs. Explain what is a normal timer and how it works. How is this timer reset?

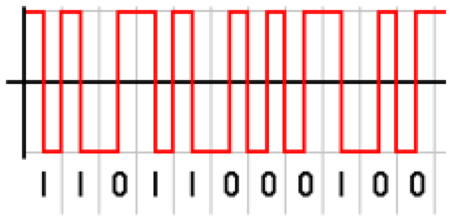
   A normal timer counts the time until the duration written in the timer is reached, as long as there is electrical continuity at its left. In this case the variable associated to the timer becomes true. The timer is reset when there is no electrical continuity at its left, meaning that the logical conditions of the elements that precede the timer in the diagram are false.

2. Consider now a drilling machine: when a **START** button is pressed, the drilling machine starts moving down (variable **GODOWN** is true) and the spindle (variable **SPINDLE**) is turned on. When the drill reaches a bottom position (variable **BOTTOM** is true) the machine stops its motion downwards (variable **GODOWN** is false), while the spindle is still on. The machine remains in such position for 10 s to make the hole, after which it moves upwards (variable **GOUP** is true). When the drill reaches an upper position (variable **UP** is true) it stops and the spindle is turned off. Program the system with a Ladder Diagram code.

   The logic system can be represented with the following ladder diagram:

```
   START   SPINDLE        GODOWN
|----| |-----|/|----+-------(L)------|
|                   |                |
|                   |     SPINDLE    |
|                   +-------(L)------|
|                                    |
|  BOTTOM           GODOWN           |
|----| |--+----------(U)-------------|
|         |                          |
|         |      +-----------+       |
|         +------| T1        |-------|
|                | 10S       |       |
|                +-----------+       |
|                                    |
|     T1             GOUP            |
|----| |-------------(L)-------------|
|                                    |
|     UP     GOUP         GOUP       |
|----| |----| |----+-----(U)---------|
|                  |                 |
|                  |   SPINDLE       |
|                  +-----(U)---------|
```

3. In a communication protocol for a digital network what are the elements defined by the physical layer? What type of line coding is represented in the following picture?



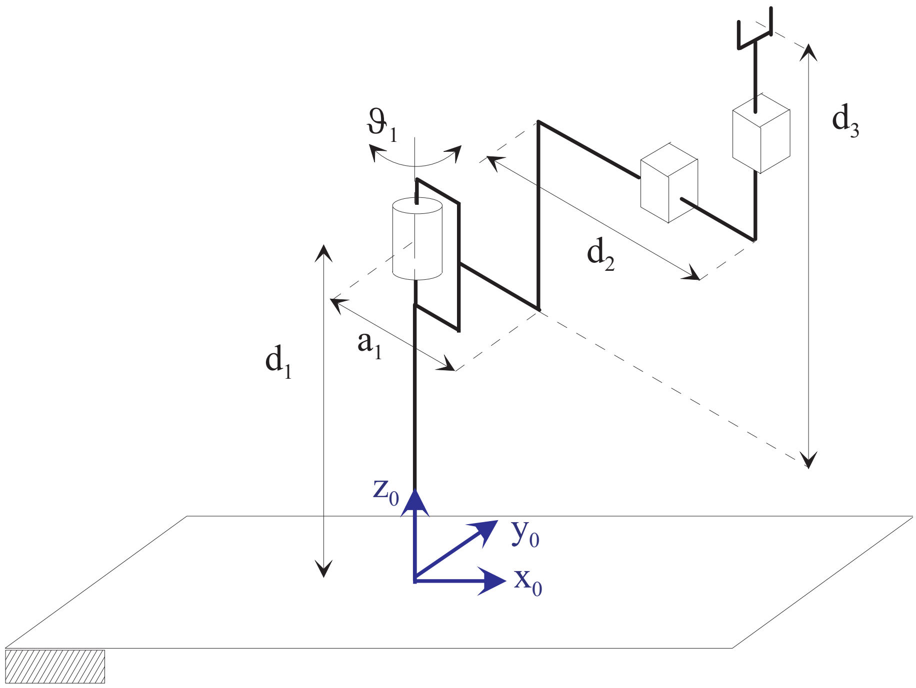
The physical layer defines the low-level properties of the protocol, including: the transmission media, the line coding, the transmission mode and the synchronization, the topology of the network. The picture shows a Manchester line coding, where 0 and 1 are represented through transitions between two levels.

4. With reference to the medium access control, explain why the CSMA/CD protocol (used in Ethernet) is not suitable for real time applications, while the token bus (or token ring) is suitable.

With the CSMA/CD protocol there is no upper limit to the time in which an element of the network can access the transmission channel, whereas with the token bus method an upper limit related to a worst case scenario can be considered.

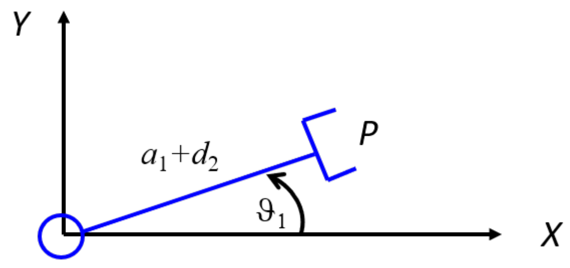
EXERCISE 3

Consider the following robot manipulator with 3 joints (rotational, prismatic and prismatic):



1. Find the expression of the direct kinematics of the robot, in terms of the position coordinates of the end effector with respect to the joint variables ϑ_1, d_2, and d_3.

   We can make reference to a projection of the manipulator on the plane x_0, y_0:

   From this view it is straightforward to write:

   $$\begin{aligned} p_x &= (a_1 + d_2)\cos(\vartheta_1) \\ p_y &= (a_1 + d_2)\sin(\vartheta_1) \end{aligned}$$

   From the 3D sketch we can conclude that:

   $$p_z = d_1 + d_3$$

2. Explain what is a homogeneous transformation matrix. For the specific manipulator of this exercise, what is the expression of the fourth coloumn of the homogeneous transformation matrix from the frame 0 to a frame with origin at the end effector?

   A homogeneous transformation matrix is a 4×4 matrix that encodes both the rotation and the translation of a frame 1 with respect to a frame 0:

   $$\mathbf{A}_1^0 = \begin{bmatrix} \mathbf{R}_1^0 & \mathbf{o}_1^0 \\ \mathbf{0} & 1 \end{bmatrix}$$

   where $\mathbf{R}_1^0$ is the rotation matrix of frame 1 with respect to frame 0 while $\mathbf{o}_1^0$ is the origin of frame 1 with respect to frame 0. Since the fourth column of the homogeneous transformation matrix represents a position vector, such column in this case is nothing else than the direct kinematics, therefore:

   $$\mathbf{A}_1^0 = \begin{bmatrix} * & * & * & (a_1 + d_2)\cos(\vartheta_1) \\ * & * & * & (a_1 + d_2)\sin(\vartheta_1) \\ * & * & * & d_1 + d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

3. Write the expression of the Jacobian of the manipulator of this exercise.

   The Jacobian takes the following expression:

$$\mathbf{J} = \begin{bmatrix} \frac{\partial p_x}{\partial \vartheta_1} & \frac{\partial p_x}{\partial d_2} & \frac{\partial p_x}{\partial d_3} \\ \frac{\partial p_y}{\partial \vartheta_1} & \frac{\partial p_y}{\partial d_2} & \frac{\partial p_y}{\partial d_3} \\ \frac{\partial p_z}{\partial \vartheta_1} & \frac{\partial p_z}{\partial d_2} & \frac{\partial p_z}{\partial d_3} \end{bmatrix} = \begin{bmatrix} -(a_1 + d_2)\sin(\vartheta_1) & \cos(\vartheta_1) & 0 \\ (a_1 + d_2)\cos(\vartheta_1) & \sin(\vartheta_1) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

4. Characterize the singularities of the manipulator of this exercise.

   Singularities are found by imposing that the Jacobian is singular:

$$\det(\mathbf{J}) = -(a_1 + d_2)\sin(\vartheta_1)^2 - (a_1 + d_2)\cos(\vartheta_1)^2 = -(a_1 + d_2)$$

   Clearly, the only situation when the Jacobian is singular is when $a_1 + d_2 = 0$.