

# Industrial Automation and Robotics

PROF. ROCCO

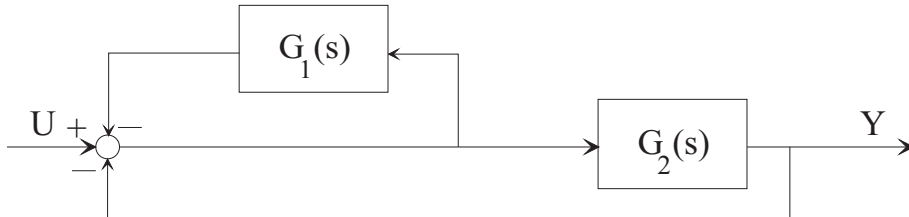
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**SOLUTION**

INDUSTRIAL AUTOMATION AND ROBOTICS  
 PROF. PAOLO ROCCO

**EXERCISE 1**

1. Consider the dynamical system described by the following block diagram:



Solve the block diagram by determining the transfer function from  $u$  to  $y$ .

The block diagram includes an inner feedback loop with unitary forward transfer function and transfer function  $G_1$  in negative feedback, whose closed-loop transfer function is  $\frac{1}{1+G_1}$ . In turn, this system, in series with  $G_2$ , is closed in a unitary negative feedback. The resulting transfer function is:

$$\frac{Y}{U} = \frac{\frac{G_2}{1+G_1}}{1 + \frac{G_2}{1+G_1}} = \frac{G_2}{1 + G_1 + G_2}$$

2. Setting  $G_1(s) = k$ ,  $G_2(s) = \frac{1}{s}$ , assign the parameter  $k$  such that the overall system has a time constant  $T = 10$ .

Substituting the expressions of  $G_1$  and  $G_2$  we obtain:

$$\frac{Y}{U} = \frac{\frac{1}{s}}{1 + k + \frac{1}{s}} = \frac{1}{s(1+k) + 1}$$

The time constant of this transfer function is:

$$T = 1 + k$$

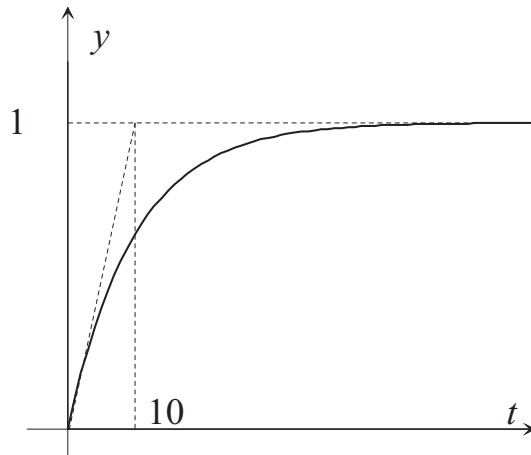
In order for this time constant to be 10,  $k$  has to be equal to 9.

3. Using the value of  $k$  found at the previous step, sketch the step response from input  $u$  to output  $y$ . What is the approximate duration of the transient?

The resulting transfer function is:

$$\frac{Y}{U} = \frac{1}{1 + 10s} = \frac{\mu}{1 + Ts}$$

with  $\mu = 1$  and  $T = 10$ . The step response of such transfer function is known and is sketched here:

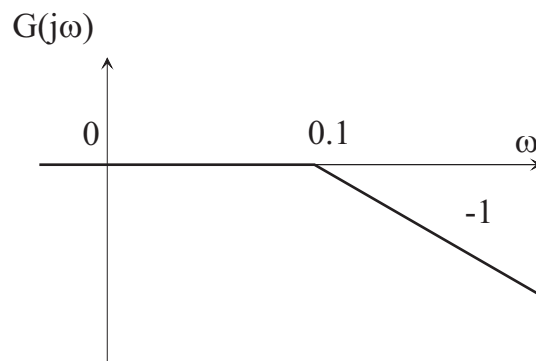


The approximate duration of the transient is 5 times the time constant  $T$ , i.e. 50 s.

- Define what a frequency response is in general. Then for the transfer function discussed in the previous step, sketch the Bode diagram of the magnitude of the frequency response.

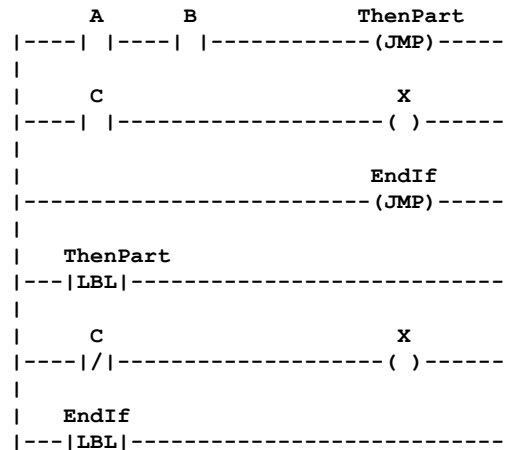
Given a transfer function  $G(s)$ , the frequency response is the function of the real variable  $\omega$  defined as  $G(j\omega)$ , for  $\omega > 0$ .

The Bode diagram of the magnitude of the frequency response can be easily sketched:



## EXERCISE 2

- Consider the following Ladder Diagram code for a PLC:



Explain what this code stands for.

This code implements an “if-then-else” structure. Specifically, the meaning is:

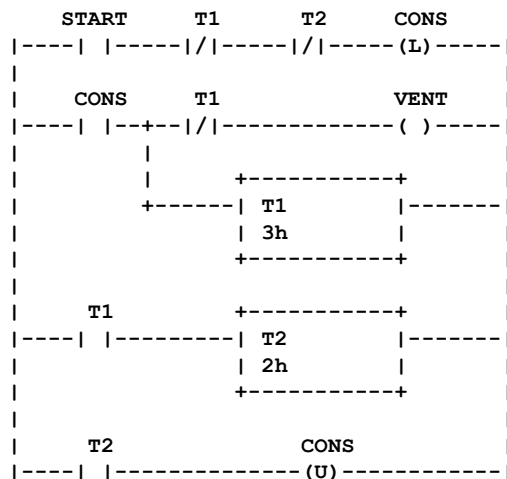
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if (A and B) then X = not(C)
else
X = C

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2. Consider now an automatic ventilation system. Pressing a button START the automatic ventilation system is activated for 3 hours. After such time interval, the ventilation system turns off and for 2 hours pressing the button START cannot activate the ventilation system. Program the system with a Ladder Diagram code.

The PLC can be programmed with this code, that uses an input variable START, an internal variable CONS, an output variable VENT and two timers:



3. Briefly explain what a hard real-time system is, providing an example.

A hard real-time systems is a system where failure to meet real time requirements can lead to serious or even catastrophic consequences. For example, a digital control system for a safety critical application may lead to serious issues in case of failures in the real time requirements.

4. A real-time operating system typically can enforce “pre-emption” of the processes. Explain what this stands for and why this feature can be beneficial for real-time purposes.

Pre-emption means that the operating system has the authority to temporarily stop a process with lower priority and assign resources to a process with higher priority, that needs to meet some real-time deadline. In this way satisfaction of real-time requirements is favoured.

### EXERCISE 3

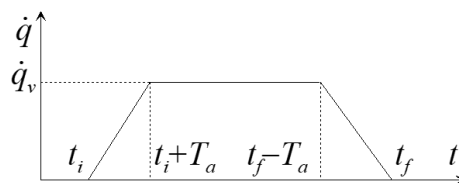
1. Explain what is the difference between the trajectory generation for a robot in the joint space and in the operational space.

When planning the trajectory in the joint space, the motion is generated for each joint independently from the other joints. There are no issues of singularities, as inverse kinematics is not needed, however the motion of the end-effector is not under control.

When the trajectory is planned in the operational space, the motion of the end effector (position and orientation) is under control, however inverse kinematics has to be run and this can lead to singularity issues.

2. Consider now the design of a joint trajectory with a trapezoidal velocity profile. The total displacement is  $h = 2$ , the total positioning time is  $T = 1s$  and the acceleration time is  $T_a = 0.2s$ . Compute the constant speed in the central part of the motion and the acceleration at the beginning of the trajectory

The total displacement is the area under the trapezoidal of the speed, and, with reference to the following picture:



can be computed as:

$$h = 0.5\dot{q}_v (T + T - 2T_a) = \dot{q}_v (T - T_a)$$

Then:

$$\dot{q}_v = \frac{h}{T - T_a} = \frac{2}{1 - 0.2} = 2.5$$

The acceleration at the beginning of the trajectory is easily computed as:

$$\ddot{q}_a = \frac{\dot{q}_v}{T_a} = 12.5$$

3. Consider now the generation of the trajectory in the operational space. Write the expression of a segment (linear Cartesian path) for the position of the end effector.

The expression of the segment is as follows:

$$\mathbf{p}(s) = \mathbf{p}_1 + \frac{s}{\|\mathbf{p}_2 - \mathbf{p}_1\|} (\mathbf{p}_2 - \mathbf{p}_1)$$

4. Suppose that you want to use the trapezoidal velocity profile previously computed for the generation of the operational space trajectory (the segment as previously discussed). Explain how this can be done. What would be the maximum linear velocity of the end-effector in this case?

The time law is assigned to the natural coordinate  $s$ :

$$\mathbf{p}(s) = \mathbf{p}_1 + \frac{s(t)}{\|\mathbf{p}_2 - \mathbf{p}_1\|} (\mathbf{p}_2 - \mathbf{p}_1)$$

The maximum velocity of the end effector is the maximum speed achieved by the time law, in our case:

$$\dot{s}_{max} = \dot{q}_v = 2.5$$