

# Industrial Automation and Robotics

PROF. ROCCO

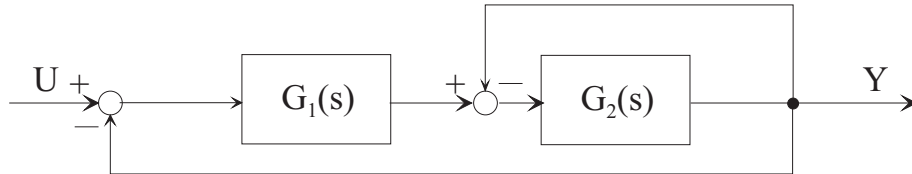
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**SOLUTION**

INDUSTRIAL AUTOMATION AND ROBOTICS  
 PROF. PAOLO ROCCO

**EXERCISE 1**

1. Consider the dynamical system described by the following block diagram:



Solve the block diagram by determining the transfer function from  $u$  to  $y$ .

The block diagram includes an inner feedback loop with forward transfer function  $G_2$  and unitary negative feedback, whose closed-loop transfer function is  $\frac{G_2}{1+G_2}$ . In turn, this system, in series with  $G_1$ , is closed in another unitary negative feedback. The resulting transfer function is:

$$\frac{Y}{U} = \frac{\frac{G_1 G_2}{1+G_2}}{1 + \frac{G_1 G_2}{1+G_2}} = \frac{G_1 G_2}{1 + G_2 + G_1 G_2}$$

2. Discuss whether it is necessary and/or sufficient that one or more of the transfer functions be asymptotically stable in order for the overall system to be asymptotically stable

Since both  $G_1$  and  $G_2$  are closed in feedback loops, it is neither necessary nor sufficient that any of them be asymptotically stable for the overall system to be asymptotically stable.

3. Setting  $G_1(s) = k$ ,  $G_2(s) = \frac{1}{1+s}$ , assign the parameter  $k$  such that the dc gain of the overall transfer function is  $\mu = 0.5$ .

Substituting the expressions of  $G_1$  and  $G_2$  we obtain:

$$\frac{Y}{U} = G(s) = \frac{k \frac{1}{1+s}}{1 + \frac{1}{1+s} + k \frac{1}{1+s}} = \frac{k}{s + 2 + k}$$

The dc gain of this transfer function is the value that it takes for  $s = 0$ :

$$\mu = G(0) = \frac{k}{2 + k}$$

which is equal to 0.5 when  $k = 2$ . In this case the transfer function is:

$$\frac{Y}{U} = G(s) = \frac{2}{s + 4}$$

- Using the value of  $k$  found at the previous step, write the equations of the dynamic system in state form that has the transfer function of the overall system obtained in the previous step.

The transfer function corresponds to a first order system of the form:

$$\begin{aligned}\dot{x} &= ax + bu \\ y &= cx\end{aligned}$$

whose transfer function is:

$$\frac{Y}{U} = \frac{bc}{s - a}$$

Comparing this transfer function with the previously obtained one, we can set  $a = -4$ ,  $b = 2$ ,  $c = 1$ . The system's equations are then:

$$\begin{aligned}\dot{x} &= -4x + 2u \\ y &= x\end{aligned}$$

## EXERCISE 2

- Explain what a discrete events system is and what is the importance of discrete events systems in the context of an industrial automation system.

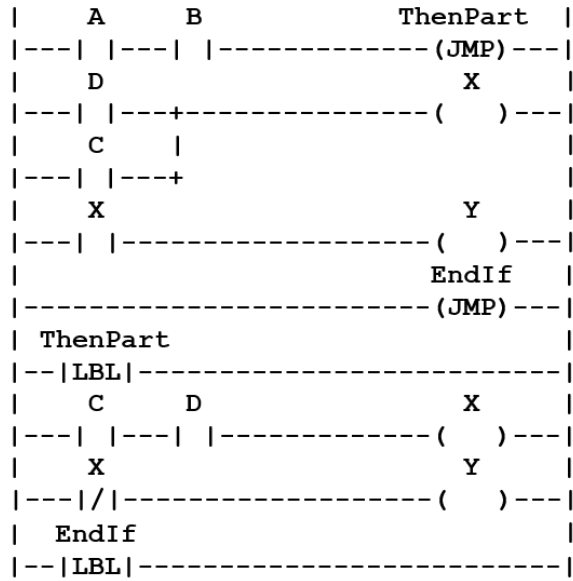
A discrete events system is a dynamic system where the state takes values only in a discrete set of values and the evolution of the state is given by the occurrence of asynchronous events. The sequence of actions typical of an industrial automation system is described by a discrete events system.

- Consider now a logical system that has to implement the function:

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if (A and B)
X = (C and D); Y = not(X);
else
X = (C or D); Y = X;
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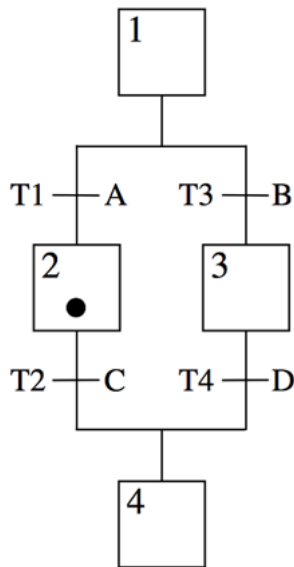
Sketch a ladder diagram that implements this logical function.

The ladder diagram can use, apart from usual elements like contacts and coils, the special **(JMP)** coil:



3. Sketch a Sequential Function Chart that implements an if-then-else programming structure and comment such sketch.

A SFC that implements an if-then-else programming structure is as follows:



The transitions A and B must be mutually exclusive .

- Consider now the interconnection of digital systems. Explain what are the advantages of a bus architecture compared to a centralized architecture. What do we mean with physical layer of a bus?

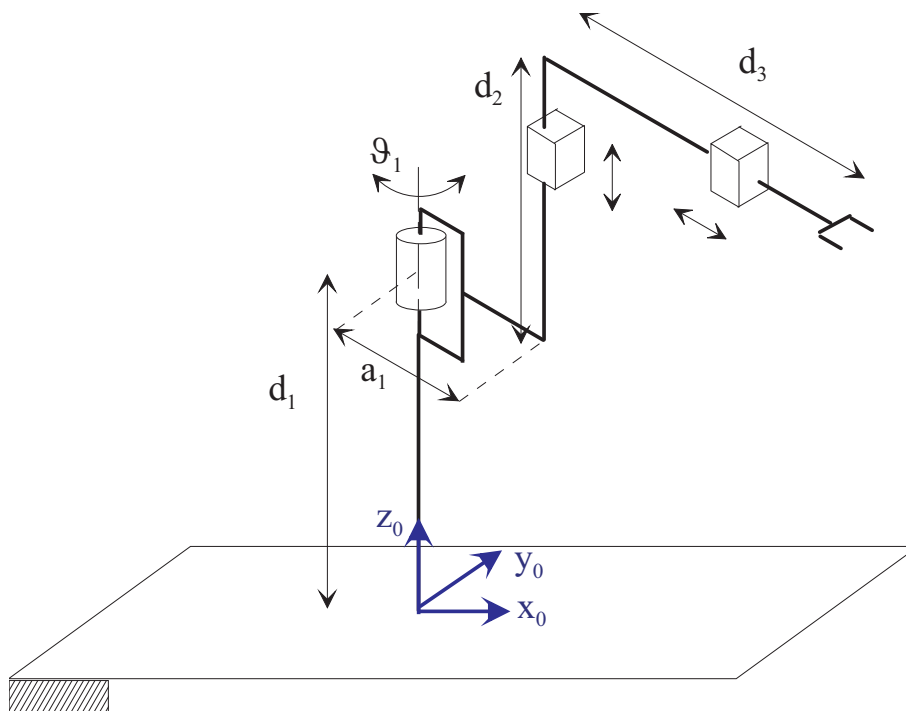
A bus architecture with digital transmission of signals allows:

- Savings on wiring costs
- Easy addition and removal of devices
- Resource sharing
- Flexibility
- Functional decentralization
- Distributed intelligence (local diagnostic features)

The physical layer of a bus defines the low-level properties, like electrical and mechanical specifications, encoding mode, transmission mode.

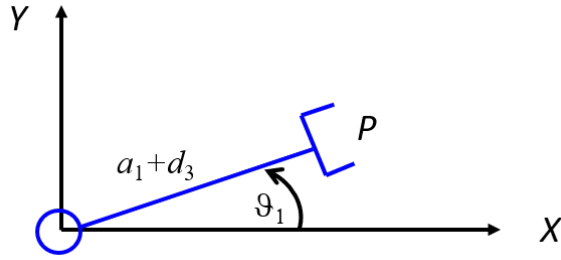
### EXERCISE 3

Consider the following robot manipulator with 3 joints (rotational, prismatic and prismatic):



- Find the expression of the direct kinematics of the robot, in terms of the position coordinates of the end effector with respect to the joint variables  $\vartheta_1$ ,  $d_2$ , and  $d_3$ .

We can make reference to a projection of the manipulator on the plane  $x_0, y_0$ :



From this view it is straightforward to write:

$$\begin{aligned} p_x &= (a_1 + d_3) \cos(\vartheta_1) \\ p_y &= (a_1 + d_3) \sin(\vartheta_1) \end{aligned}$$

From the 3D sketch we can conclude that:

$$p_z = d_1 + d_2$$

- Write the expression of the Jacobian of the manipulator of this exercise.

The Jacobian takes the following expression:

$$\mathbf{J} = \begin{bmatrix} \frac{\partial p_x}{\partial \vartheta_1} & \frac{\partial p_x}{\partial d_2} & \frac{\partial p_x}{\partial d_3} \\ \frac{\partial p_y}{\partial \vartheta_1} & \frac{\partial p_y}{\partial d_2} & \frac{\partial p_y}{\partial d_3} \\ \frac{\partial p_z}{\partial \vartheta_1} & \frac{\partial p_z}{\partial d_2} & \frac{\partial p_z}{\partial d_3} \end{bmatrix} = \begin{bmatrix} -(a_1 + d_3) \sin(\vartheta_1) & 0 & \cos(\vartheta_1) \\ (a_1 + d_3) \cos(\vartheta_1) & 0 & \sin(\vartheta_1) \\ 0 & 1 & 0 \end{bmatrix}$$

- Characterize the singularities of the manipulator of this exercise.

Singularities are found by imposing that the Jacobian is singular:

$$\det(\mathbf{J}) = -\left(- (a_1 + d_3) \sin(\vartheta_1)^2 - (a_1 + d_3) \cos(\vartheta_1)^2\right) = a_1 + d_3$$

Clearly, the only situation when the Jacobian is singular is when  $d_3 = -a_1$ .

- Consider now the safe interaction between a robot and a human. Explain what is the speed and separation monitoring safe interaction mode, writing down the inequality that characterizes such interaction mode.

In the speed and separation monitoring, the speed of the robot has to be related to the perceived distance between the robot and the human, so as to leave enough time to the robot to come to a complete stop before hitting the human. The inequality is written as:

$$D(t_0) - v_R(T_R + T_B) - v_H(T_R + T_B) \geq S$$

where  $v_R$  is the robot speed,  $v_H$  is the human speed,  $T_R$  is the robot controller reaction time,  $T_B$  is the robot stopping time,  $S$  is the minimum separation distance,  $D(t_0)$  is the separation distance at time  $t_0$ .